ex 68 $z = 2+i$

z est le conjugué de z'

$\Leftrightarrow z = \overline{z'} \Leftrightarrow \overline{z} = \overline{\overline{z'}} \Leftrightarrow \overline{z} = z'$

on a donc $z' = 2 - i = 2 - 1i$ $\quad Re(z') = 2 \quad Im(z') = -1$

$z^2 = (2+i)^2$

$= 4 + 4i + i^2 \quad Re(z^2) = 3 \quad Im(z^2) = 4$

$= 3 + 4i$

ex 39

$\boxed{z\overline{z} = a^2 + b^2}$

1°) $\frac{1}{z} = \frac{1}{-2+3i} = \frac{1(-2-3i)}{(-2+3i)(-2-3i)}$

$= \frac{-2-3i}{(-2)^2+3^2} = \frac{-2-3i}{13} = -\frac{2}{13} - \frac{3}{13}i$

2°) $-\frac{1}{z'} = \frac{-1}{7+4i} = \frac{-1(7-4i)}{(7+4i)(7-4i)}$

$= \frac{-7+4i}{7^2+4^2} = -\frac{7}{65} + \frac{4}{65}i$

3°) $\frac{z}{z'} = \frac{-2+3i}{7+4i} = \frac{(-2+3i)(7-4i)}{7^2+4^2}$

$= \frac{-14+8i+21i-12i^2}{65} = \frac{-2}{65} + \frac{29}{65}i$

4°) $\frac{2}{z^2} = \frac{2}{(-2+3i)^2} = \frac{2}{(-2)^2+2\times(-2)\times 3i+(3i)^2}$

$= \frac{2}{4-12i+9i^2} = \frac{2}{-5-12i} = \frac{2(-5+12i)}{(-5)^2+(-12)^2}$

$= \frac{-10+24i}{169} = \frac{-10}{169} + \frac{24}{169}i$

5°) $\frac{2+z}{7-z'} = \frac{2-2+3i}{7-(7+4i)} = \frac{3i}{-4i} = -\frac{3}{4}$

6°) $\frac{1-z}{1+z'} = \frac{1-(-2+3i)}{1+7+4i} = \frac{3-3i}{8+4i} = \frac{(3-3i)(8-4i)}{8^2+4^2}$

$= \frac{24-12i-24i+12i^2}{80} = \frac{12}{80} - \frac{36}{80}i = \frac{3}{20} - \frac{9}{20}i$

ex 40

1°) $\frac{2}{z'} = \frac{2}{-1+2i} = \frac{2(-1-2i)}{(-1)^2+2^2} = \frac{-2-4i}{5} = -\frac{2}{5} - \frac{4}{5}i$

2°) $-\frac{3}{\bar{z}} = \frac{-3}{\bar{z}} = \frac{-3}{4-2i} = \frac{-3(4+2i)}{4^2+(-2)^2} = \frac{-12}{20} - \frac{6}{20}i = -\frac{3}{5} - \frac{3}{10}i$

3°) $\frac{z}{z'} = \frac{4-2i}{-1+2i} = \frac{(4-2i)(-1-2i)}{(-1)^2+2^2} = \frac{-4-8i+2i+4i^2}{5}$

$= \frac{-8-6i}{5} = -\frac{8}{5} - \frac{6}{5}i$

4°) $-\frac{3}{z^2} = \frac{-3}{(4-2i)^2} = \frac{-3}{4^2-2\times4\times2i+(2i)^2} = \frac{-3}{12-16i}$

$= \frac{-3(12-16i)}{12^2+(-16)^2} = -\frac{36}{400} + \frac{48}{400}i = -\frac{9}{100} + \frac{3}{25}i$

5°) $\frac{2i+z}{1+z'} = \frac{2i+4-2i}{1-1+2i} = \frac{4}{2i} = \frac{4(-2i)}{0^2+2^2}$ $2i = 0+2i$

$= -2i$

6°) $\frac{2-z}{2+z'} = \frac{2-(4-2i)}{2+(-1+2i)} = \frac{-2+2i}{1+2i} = \frac{(-2+2i)(1-2i)}{1^2+2^2}$

$= \frac{-2+4i+2i-4i^2}{5} = \frac{2}{5} + \frac{6}{5}i$

ex 41

1°) $2 + \frac{1}{i} = 2 + \frac{1\times(-i)}{0^2+1^2}$ $i = 0+1i$

$= 2-i$

2°) $-3 - \frac{4}{2+i} = -3 - \frac{4(2-i)}{2^2+1^2} = -3 - \frac{(8-4i)}{5}$

$= -3 - \frac{8}{5} + \frac{4}{5}i$ Attention au signe "–" devant le trait de fraction

$= -\frac{23}{5} + \frac{4}{5}i$

3°) $i - \frac{1}{2i} = i - \frac{1(-2i)}{0^2+2^2} = i + \frac{2i}{4} = \frac{3}{2}i$

4°) $\frac{2-3i}{5-i} = \frac{(2-3i)(5+i)}{5^2+(-1)^2} = \frac{10+2i-15i+3i^2}{26} = \frac{7}{26} - \frac{13}{26}i = \frac{7}{26} - \frac{1}{2}i$

5°) $\frac{3-2i}{3+2i} = \frac{(3-2i)^2}{3^2+2^2} = \frac{9-2\times3\times2i+(2i)^2}{13} = \frac{5}{13} - \frac{12}{13}i$

ex 73 :

1°) $z_1 \times z_2 = (1-i)(2i+3) = 2i+3-2i^2-3i = 5-i$

2°) $z_1^2 = (1-i)^2 = 1^2-2\times1\times i+i^2 = -2i$

3°) $\frac{z_1}{z_2} = \frac{1-i}{2i+3} = \frac{1-i}{3+2i} = \frac{(1-i)(3-2i)}{3^2+2^2}$

$= \frac{3-2i-3i+2i^2}{13} = \frac{1}{13} - \frac{5}{13}i$

4°) $3z_1 - 2 = 3(1-i)-2 = 1-3i$

5°) $z_2 + \overline{z_1} \times \overline{z_2}$

$= 2i+3 + \overline{1-i} \times \overline{2i+3}$

$= 2i+3 + (1+i) \times \overline{3+2i}$

$= 2i+3 + (1+i)(3-2i)$

$= 2i+3 + 3-2i+3i-2i^2$

$= \cancel{2i}+3+3\cancel{-2i}+3i+2$

$= 8+3i$