

Propriétés algébriques - Opérations et conjugués

Ex 63 :

$$z = 5 + 3i \quad \text{et} \quad z' = 2 - i$$

$$\text{donc } \bar{z} = 5 - 3i \quad \bar{z}' = 2 + i$$

$$1^{\circ}) \overline{z+z'} = \bar{z} + \bar{z}' = 5 - 3i + 2 + i = 7 - 2i$$

$$2^{\circ}) \overline{z-z'} = \bar{z} - \bar{z}' = (5 - 3i) - (2 + i) = 5 - 3i - 2 - i = 3 - 4i$$

$$3^{\circ}) \overline{4z} = \overline{4z} = 4\bar{z} = 4(5 - 3i) = 20 - 12i$$

$$\begin{aligned} 4^{\circ}) \overline{2z+3z'} &= 2\bar{z} + 3\bar{z}' = 2(5 - 3i) + 3(2 + i) \\ &= 10 - 6i + 6 + 3i = 16 - 3i \end{aligned}$$

$$\begin{aligned} 5^{\circ}) \overline{z z'} &= \bar{z} \bar{z}' = (5 - 3i)(2 + i) \\ &= 10 + 5i - 6i - 3i^2 = 10 - i + 3 = 13 - i \end{aligned}$$

$$\begin{aligned} 6^{\circ}) \overline{z^2} &= \bar{z}^2 = (5 - 3i)^2 = 5^2 - 2 \times 5 \times 3i + (3i)^2 \\ &= 25 - 30i + 9i^2 = 25 - 30i - 9 = 16 - 30i \end{aligned}$$

$$\begin{aligned} 7^{\circ}) \frac{1}{\bar{z}} &= \frac{1}{\bar{z}} = \frac{1}{5 - 3i} = \frac{1(5 + 3i)}{(5 - 3i)(5 + 3i)} \quad \text{avec } \bar{\bar{z}} = a^2 + b^2 \\ &= \frac{5 + 3i}{5^2 + (-3)^2} = \frac{5 + 3i}{34} \\ &= \frac{5}{34} + \frac{3}{34}i \end{aligned}$$

$$\begin{aligned} 8^{\circ}) \frac{\bar{z}}{z'} &= \frac{\bar{z}}{z'} = \frac{5 - 3i}{2 + i} = \frac{(5 - 3i)(2 - i)}{(2 + i)(2 - i)} \\ &= \frac{10 - 5i - 6i + 3i^2}{2^2 + 1^2} = \frac{10 - 11i - 3}{5} \\ &= \frac{7}{5} - \frac{11}{5}i \end{aligned}$$

Ex 6h:

$$z = \frac{(1+i)(2+i)}{3i(5-i)}$$

$$\text{donc } \bar{z} = \frac{(1-i)(2-i)}{(-3i)(5+i)}$$

$$= \frac{2-i-2i+i^2}{-15i-3i^2}$$

$$= \frac{2-3i-1}{-15i+3}$$

$$= \frac{1-3i}{3-15i} = \frac{(1-3i)(3+15i)}{(3-15i)(3+15i)}$$

$$= \frac{3+15i-9i-45i^2}{3^2+(-15)^2}$$

$$= \frac{3+6i+45}{234}$$

$$= \frac{48}{234} + \frac{6}{234}i$$