Opérations sur les fonctions dérivées

Ex 60 :

1) $f(x) = 5x^3 - 7x + 2$ $\quad \forall x \in \mathbb{R} \quad f'(x) = 15x^2 - 7$

2) $f(t) = -7t^2 - \frac{3}{t} + 5 = -7t^2 - 3 \times \frac{1}{t} + 5$

$\forall x \in I \quad f'(t) = -14t - 3 \times \left(-\frac{1}{t^2}\right) = -14t + \frac{3}{t^2} = \frac{-14t^3 + 3}{t^2}$

3°) $f(x) = (2x-3)^2(x^2+1)$ $\qquad (uv)' = u'v + uv'$

$u(x) = (2x-3)^2 = 4x^2 - 12x + 9 \qquad u'(x) = 8x - 12$

$v(x) = x^2 + 1 \qquad v'(x) = 2x$

$\forall x \in I \quad f'(x) = (8x-12)(x^2+1) + (2x-3)^2 \times 2x$

4°) $f(a) = \frac{4a^5 - 10a^2 + 3}{2a} = 2a^4 - 5a + \frac{3}{2a}$ $\qquad$ Bonne méthode

$u(a) = 4a^5 - 10a^2 + 3 \qquad u'(a) = 20a^4 - 20a$

$v(a) = 2a \qquad v'(a) = 2$

$f'(a) = 8a^3 - 5 + \frac{3}{2} \times \left(\frac{-1}{a^2}\right) = 8a^3 - 5 - \frac{3}{2a^2}$

$\forall t \in I \quad f'(a) = \frac{(20a^4 - 20a)2a - (4a^5 - 10a^2 + 3)2}{2a}$

↳ mauvaise méthode dans ce cas là

Ex 61 :

1°) $f(t) = 2t^2 + \frac{5}{t^2} - 5$ $\qquad \forall t \in I \quad f'(t) = 4t + 5 \times \left(\frac{-2}{t^3}\right) + 0 = 4t - \frac{10}{t^3}$

2°) $f(t) = 3 - 4t - \frac{2}{3t^2} = 3 - 4t - \frac{2}{3} \times \frac{1}{t^2}$

$\forall t \in I \quad f'(t) = -4 - \frac{2}{3} \times \left(\frac{-2}{t^3}\right) = -4 + \frac{4}{3t^3}$

3°) $f(x) = 6\sqrt{x} - \frac{3}{10x^5} = 6\sqrt{x} - \frac{3}{10} \times \frac{1}{x^5}$

$\forall x \in I \quad f'(x) = 6 \times \frac{1}{2\sqrt{x}} - \frac{3}{10} \times \frac{-5}{x^6} = \frac{3}{\sqrt{x}} + \frac{3}{2x^6}$

4°) $f(a) = \frac{-3a^4 + a^2 - 1}{8a^4} = -\frac{3a^4}{8a^4} + \frac{a^2}{8a^4} - \frac{1}{8a^4} = -\frac{3}{8} + \frac{1}{8a^2} - \frac{1}{8a^4}$

$\forall a \in I \quad f'(a) = 0 + \frac{1}{8} \times \frac{-2}{a^3} - \frac{1}{8} \times \frac{-4}{a^5} = \frac{-2}{8a^3} + \frac{4}{8a^5}$

ex 62 : 1°) $f(x) = (x^2+1)\sqrt{x}$ $\quad (uv)' = u'v + uv'$

$u(x) = x^2+1 \qquad u'(x) = 2x$

$v(x) = \sqrt{x} \qquad v'(x) = \dfrac{1}{2\sqrt{x}}$

$\forall x \in I \quad f'(x) = 2x\sqrt{x} + \dfrac{x^2+1}{2\sqrt{x}} = \dfrac{2x\sqrt{x} \times 2\sqrt{x} + x^2 + 1}{2\sqrt{x}}$

on utilise ici $\sqrt{x} \times \sqrt{x} = x$ $\quad = \dfrac{4x^2+x^2+1}{2\sqrt{x}} = \dfrac{5x^2+1}{2\sqrt{x}}$

- 2°) $f(x) = \dfrac{4}{x^2+2x+3} = 4 \times \dfrac{1}{x^2+2x+3}$ $\quad \left(\dfrac{1}{u}\right)' = -\dfrac{u'}{u^2}$

$u(x) = x^2+2x+3 \qquad u'(x) = 2x+2$

$\forall x \in I \quad f'(x) = 4 \times \dfrac{-(2x+2)}{(x^2+2x+3)^2} = \dfrac{-4(2x+2)}{(x^2+2x+3)^2}$

- 3°) $f(t) = \dfrac{4t-2}{2t^2+3t}$ $\quad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$u(t) = 4t-2 \qquad u'(t) = 4$

$v(t) = 2t^2+3t \qquad v'(t) = 4t+3$

$\forall t \in I \quad f'(t) = \dfrac{4(2t^2+3t) - (4t-2)(4t+3)}{(2t^2+3t)^2}$

$= \dfrac{8t^2 + 12t - (16t^2 + 12t - 8t - 6)}{(2t^2+3t)^2}$

$= \dfrac{-8t^2 + 8t + 6}{(2t^2+3t)^2}$

- 4°) $f(x) = \dfrac{3\sqrt{x}}{x+1}$ $\quad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$u(x) = 3\sqrt{x} \qquad u'(x) = \dfrac{3}{2\sqrt{x}}$

$v(x) = x+1 \qquad v'(x) = 1$

$\forall x \in I \quad f'(x) = \dfrac{\dfrac{3}{2\sqrt{x}} \times (x+1) - 3\sqrt{x} \times 1}{(x+1)^2}$

$= \dfrac{\dfrac{3(x+1) - 3\sqrt{x} \times 2\sqrt{x}}{2\sqrt{x}}}{(x+1)^2}$

$= \dfrac{\dfrac{3x+3-6x}{2\sqrt{x}}}{(x+1)^2} = \dfrac{-3x+3}{2\sqrt{x}\,(x+1)^2}$

ex 63

1°) $f(x) = 2x + \frac{5x^2+2x}{x^2+1}$ $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$u(x) = 5x^2+2x$ $u'(x) = 10x+2$

$v(x) = x^2+1$ $v'(x) = 2x$

$\forall x \in I \quad f'(x) = 2 + \frac{(10x+2)(x^2+1) - (5x^2+2x)2x}{(x^2+1)^2}$

2°) $f(x) = 4x^2 + 3x\sqrt{x}$ → $(uv)' = u'v + uv'$

$u(x) = 3x$ $u'(x) = 3$

$v(x) = \sqrt{x}$ $v'(x) = \frac{1}{2\sqrt{x}}$

$\forall x \in I \quad f'(x) = 8x + 3\sqrt{x} + \frac{3x}{2\sqrt{x}}$ $\frac{x}{\sqrt{x}} = \frac{\sqrt{x}\times\sqrt{x}}{\sqrt{x}} = \sqrt{x}$

$= 8x + 3\sqrt{x} + \frac{3}{2}\sqrt{x} = 8x + \frac{9}{2}\sqrt{x}$

3°) $f(t) = \frac{2}{\sqrt{t}} + 3\sqrt{t}$ $\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$

$v(t) = \sqrt{t}$ $v'(t) = \frac{1}{2\sqrt{t}}$

$\forall t \in I \quad f'(t) = 2 \times \frac{-\frac{1}{2\sqrt{t}}}{(\sqrt{t})^2} + 3 \times \frac{1}{2\sqrt{t}} = -\frac{2}{2t\sqrt{t}} + \frac{3}{2\sqrt{t}}$

4°) $f(a) = \frac{a^3-2a}{5a^3-2a+1}$ $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$u(a) = a^3-2a$ $u'(a) = 3a^2-2$

$v(a) = 5a^3-2a+1$ $v'(a) = 15a^2-2$

$\forall a \in I \quad f'(a) = \frac{(3a^2-2)(5a^3-2a+1) - (a^3-2a)(15a^2-2)}{(5a^3-2a+1)^2}$

$= \frac{15a^5 - 6a^3 + 3a^2 - 10a^3 + 4a - 2 - (15a^5 - 2a^3 - 30a^3 + 4a)}{(5a^3-2a+1)^2}$

$= \frac{16a^3 + 3a^2 - 2}{(5a^3-2a+1)^2}$