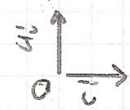
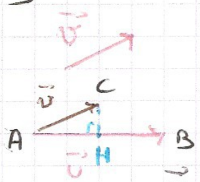
Exercice : Calculez le produit scalaire $\vec{u} \cdot \vec{v}$ dans chacun des cas suivants

Le plan est muni du repère orthonormal

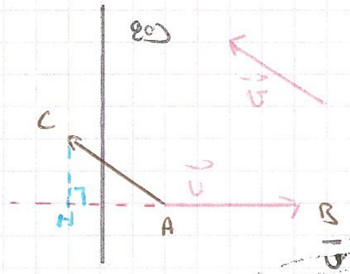


1°)



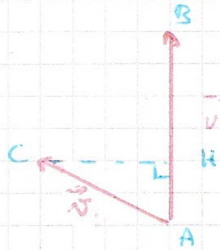
$\vec{u} \cdot \vec{v} = \vec{AB} \cdot \vec{AH}$
$= AB \times AH$
$= 2 \times 1 = 2$

2°)



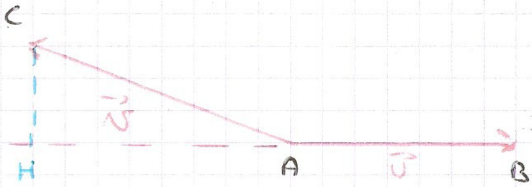
$\vec{u} \cdot \vec{v} = \vec{AB} \cdot \vec{AH}$
$= - AB \times AH$
$= -2 \times 1{,}5 = -3$

3°)



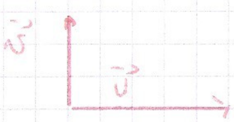
$\vec{u} \cdot \vec{v} = AB \times AH$
$= 3 \times 1 = 3$

4°)



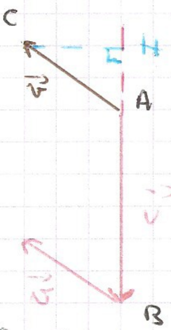
$\vec{u} \cdot \vec{v} = -AB \times AH$
$= -3{,}5 \times 4 = -14$

5°)



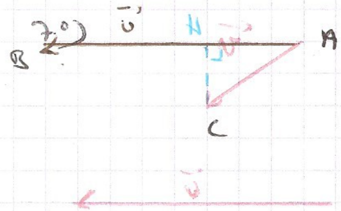
$\vec{u} \cdot \vec{v} = 0$
car $\vec{u}$ et $\vec{v}$ sont orthogonaux

6°)



$\vec{u} \cdot \vec{v} = -AB \times AH$
$= -3 \times 1 = -3$

7°)

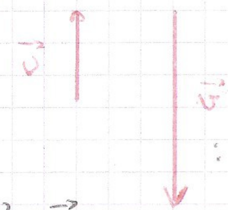


$\vec{u} \cdot \vec{v} = AB \times AH$
$= 4 \times 1{,}5 = 6$

8°)

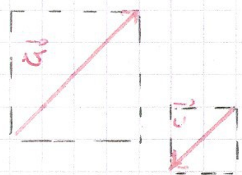
$\vec{u} \cdot \vec{v} = \|\vec{u}\| \times \|\vec{v}\|$
$= 2 \times 4 = 8$

9°)



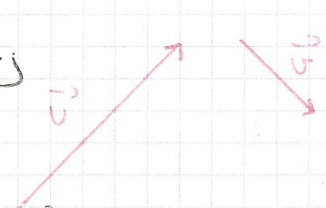
$\vec{u} \cdot \vec{v} = -\|\vec{u}\| \times \|\vec{v}\|$
$= -1{,}5 \times 3 = -4{,}5$

10°)



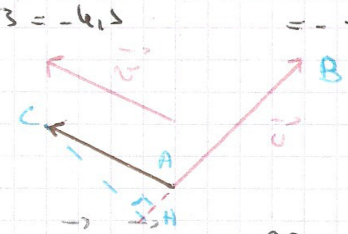
$\vec{u} \cdot \vec{v} = -\|\vec{u}\| \times \|\vec{v}\|$
$= -1\sqrt{2} \times 2\sqrt{2} = -4$
(diagonale d'un carré)

11°)



$\vec{u} \cdot \vec{v} = 0$
car $\vec{u}$ et $\vec{v}$ sont orthogonaux

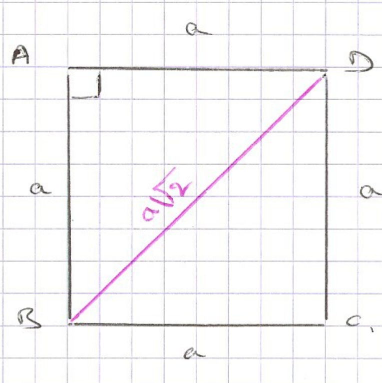
12°)



$\vec{u} \cdot \vec{v} = -AB \times AH$
$= -2\sqrt{2} \times \frac{1}{2}\sqrt{2} = -2$
(diagonale d'un carré)

Rappel :

Diagonale d'un carré de côté a



ABCD est un carré de côté a.

On veut déterminer en fonction de a la longueur des diagonales.

Dans le triangle ABD rectangle en A, on utilise la propriété de Pythagore

$BD^2 = AB^2 + AD^2$

$\Leftrightarrow BD^2 = a^2 + a^2$

$\Leftrightarrow BD^2 = 2a^2$

$\Leftrightarrow BD = \sqrt{2a^2}$

$\Leftrightarrow BD = a\sqrt{2}$

propriété :

$\sqrt{a^2b} = |a|\sqrt{b}$

ou $\sqrt{a^2b} = a\sqrt{b}$ si $a \geq 0$

Propriété : la diagonale d'un carré de côté a a pour longueur $a\sqrt{2}$