

CALCULS NUMÉRIQUES - Identités remarquables + développement

Ex 24 Développer $(a-b)^2 = a^2 - 2ab + b^2$

$$A = (3n-4)^2 = (3n)^2 - 2 \times 3n \times 4 + 4^2 = 9n^2 - 24n + 16$$

$$B = 2(2n+3)(-n+4)$$

$$= (4n+6)(-n+4)$$

$$= -4n^2 + 16n - 6n + 24 = -4n^2 + 10n + 24$$

$$C = (2n+1)^2 + (n-2)(2n+3)$$

$$= (2n)^2 + 2 \times 2n \times 1 + 1^2 + (2n^2 + 3n - 4n - 6)$$

$$= 4n^2 + 4n + 1 + 2n^2 - n - 6$$

$$= 6n^2 + 3n - 5$$

Ex 25 Développer

$$A(n) = (3n-5)^2 + 2n(n+6)$$

$$= (3n)^2 - 2 \times 3n \times 5 + 5^2 + 2n^2 + 12n$$

$$= 9n^2 - 30n + 25 + 2n^2 + 12n$$

$$= 11n^2 - 18n + 25$$

$$B(n) = (2n+1)(3n-8) - (n+4)^2$$

$$= 6n^2 - 16n + 3n - 8 - (n^2 + 2 \times n \times 4 + 4^2)$$

$$= 6n^2 - 13n - 8 - n^2 - 8n - 16$$

$$= 5n^2 - 21n - 24$$

$$C(n) = (n-5)(n+5) + (n+5)^2$$

$$(a-b)(a+b) = a^2 - b^2 \quad \Rightarrow \quad n^2 - 5^2 + n^2 + 2 \times n \times 5 + 5^2$$

$$= n^2 - 25 + n^2 + 10n + 25 = 2n^2 + 10n$$

$$D(n) = (2n+8)^2 - (n+1)(n+2)$$

$$= (2n)^2 + 2 \times 2n \times 8 + 8^2 - (n^2 + 2n + n + 2)$$

$$= 4n^2 + 32n + 64 - n^2 - 3n - 2$$

$$= 3n^2 + 29n + 62$$

Ex 31. Factorisieren

$$a^2 - b^2 = (a-b)(a+b)$$

$$A(n) = 49n^2 - 25$$

$$= (7n)^2 - 5^2 = (7n-5)(7n+5)$$

$$B(n) = 1 - 4n^2 = 1^2 - (2n)^2 = (1-2n)(1+2n)$$

$$C(n) = (n+1)^2 - 25n^2$$

$$= (n+1)^2 - (5n)^2$$

$$= [(n+1) - (5n)][(n+1) + (5n)]$$

$$= (-4n+1)(6n+1)$$

$$D(n) = 100n^2 - (2n-3)^2$$

$$= (10n)^2 - (2n-3)^2$$

$$= (10n - (2n-3))(10n + (2n-3))$$

$$= (8n+3)(12n-3)$$

Ex 24. Developpen

$$A = (3n-5)^2 = (3n)^2 - 2 \times 3n \times 5 + 5^2 = 9n^2 - 30n + 25$$

$$B = (-3n+2)(2n-4)$$

$$= -6n^2 + 12n + 4n - 8 = -6n^2 + 16n - 8$$

$$C = 2(n+1)(3n-1)$$

$$= (2n+2)(3n-1) = 6n^2 - 2n + 6n - 2$$

$$= 6n^2 + 4n - 2$$

ex 25: Factoriser

$$\begin{aligned}A &= (3n+1)^2 + 2(3n+1)(2n-5) \\&= (3n+1)(3n+1) + 2(3n+1)(2n-5) \\&= (3n+1)((3n+1) + 2(2n-5)) \\&= (3n+1)(3n-9)\end{aligned}$$

$$\begin{aligned}B &= (2n+5)^2 - (2n+5)(n+7) \\&= (2n+5)(2n+5) - (2n+5)(n+7) \\&= (2n+5)((2n+5) - (n+7)) \\&= (2n+5)(n-2)\end{aligned}$$

$$\begin{aligned}C &= (2n+1)^2 - 16 = (2n+1)^2 - 4^2 \\&= ((2n+1)-4)((2n+1)+4) = (2n-3)(2n+5)\end{aligned}$$

ex 26: Factoriser

$$A = 9n^2 - 36 = (3n)^2 - 6^2 = (3n-6)(3n+6)$$

$$\begin{aligned}B &= (2n+1)^2 - (n+5)^2 \\&= ((2n+1) - (n+5))((2n+1) + (n+5)) \\&= (n-4)(3n+6)\end{aligned}$$

$$\begin{aligned}C &= 4(n+1)^2 - 81(n-2)^2 \\&= [2(n+1)]^2 - [9(n-2)]^2 \\&= (2(n+1) - 9(n-2))(2(n+1) + 9(n-2)) \\&= (2n+2 - 9n+18)(2n+2 + 9n-18) \\&= (-7n+20)(11n-16)\end{aligned}$$

ex 21 Développer

$$A(n) = (3n+1)^2 = (3n)^2 + 2 \times 3n \times 1 + 1^2 = 9n^2 + 6n + 1$$

$$B(n) = (n-4)^2 = n^2 - 2 \times n \times 4 + 4^2 = n^2 - 8n + 16$$

ex 22: Factoriser

$$A(n) = (n-2)(n+3) - 5n(n-2)$$

$$= (n-2) \left((n+3) - 5n \right) = (n-2)(-4n+3)$$

$$B(n) = (4n-5)^2 - (n+2)(4n-5)$$

$$= (4n-5)(4n-5) - (n+2)(4n-5)$$

$$= (4n-5) \left((4n-5) - (n+2) \right) = (4n-5)(3n-7)$$

Ex 23 Factoriser

$$A(n) = 25n^2 - 36 = (5n)^2 - 6^2 = (5n-6)(5n+6)$$

$$B(n) = (2n-5)^2 - 4$$

$$= (2n-5)^2 - 2^2 = \left((2n-5) - 2 \right) \left((2n-5) + 2 \right)$$

$$= (2n-7)(2n-3)$$

Ex 32: Développer

$$A = (2n+5)^2 = (2n)^2 + 2 \times 2n \times 5 + 5^2 = 4n^2 + 20n + 25$$

$$B = (n+3)(2n-7) = 2n^2 - 7n + 6n - 21 = 2n^2 - n - 21$$

Ex 23: Développer

$$A = (3n+5)^2 + (3n+5)(2n-8) \quad \text{à comparer avec A et B}$$

$$= (3n)^2 + 2 \times 3n \times 5 + 5^2 + 6n^2 - 46n + 40n - 40$$

6ch 36 avec ou sans sign
différente.

$$= 9n^2 + 30n + 25 + 6n^2 - 46n - 40$$

$$= 15n^2 + 16n - 15$$

$$B = (2n+3)^2 - (n+1)(n+7) = 4n^2 + 12n + 9 - (n^2 + 7n + n + 7)$$

$$= 3n^2 + 4n + 2$$

Ex 34: Factorisieren

$$1^o) 36n^2 - 4 = (6n)^2 - 2^2 = (6n-2)(6n+2)$$

$$\begin{aligned} 2^o) B &= (3n+5)^2 + (3n+5)(2n-8) \\ &= (3n+5)((3n+5) + (2n-8)) \\ &= (3n+5)(5n-3) \end{aligned}$$

$$\begin{aligned} 3^o) C &= (n+2)^2 - (5n+1)(n+2) \\ &= (n+2)((n+2) - (5n+1)) \\ &= (n+2)(-4n+1) \end{aligned}$$

ex 35 Factorisieren

$$\begin{aligned} A &= (4n+3)^2 - (3n+1)^2 \\ &= ((4n+3) - (3n+1))((4n+3) + (3n+1)) \\ &= (n+2)(7n+4) \end{aligned}$$

$$\begin{aligned} B &= (n+1)(2n-3) + 2(n+1)(n+3) \\ &= (n+1)((2n-3) + 2(n+3)) \\ &= (n+1)(4n+3) \end{aligned}$$

$$\begin{aligned} C &= (3n-2)(n+2) - 3(n+2)(2n+1) \\ &= (n+2)((3n-2) - 3(2n+1)) \\ &= (n+2)(3n-2-6n-3) \\ &= (n+2)(-3n-5) \end{aligned}$$